(even at a small Ra). This increase in the strength of the convective flow can also be observed in Fig. 2.

To summarize, the flow and temperature fields are significantly modified by the inclusion of mass transfer effects. In the presence of a concentration gradient, flow can be either aided or retarded, depending on the sign of the buoyancy ratio N. The Lewis number is observed to have a stronger influence on the concentration field than it does on the flow and temperature fields. In addition, it amplifies the results produced by the buoyancy ratio.

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Radiative configuration factors from cylinders to coaxial axisymmetric bodies

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INTRODUCTION

MANY practical engineering applications in radiative heat transfer require the evaluation of geometric configuration factors between a cylinder and a coaxial axisymmetric body, such as pipe exhaust systems (including a rocket and its plume) and annular radiative fins. Although view factors for such geometries resist closed-form solutions, the number of required integrations for a numerical calculation could be reduced substantially if analytic expressions are obtained for the configuration factors between differential elements of the axisymmetric body and the cylinder. In this note, exact solutions are derived for shape factors between differential elements of arbitrary orientation and cylinders. Using these derivations, we illustrate the calculation of view factors between cylinders and general coaxial bodies via a method in which only a single numerical integration need be performed [1].

View factor from a differential element to a cylinder

The configuration shown in Fig. 1 depicts a differential area dA_2 and a cylinder Cy. The unit normal vector to dA_2 lies in the y–z plane. If the angle θ is less than $\tan^{-1}[HP/(P^2-1)]$, where $H = h/r$ and $P = p/r$ are the dimensionless cylinder height and distance from the differential element to the axis of symmetry, respectively, the contour of the section of the cylinder which is visible to the differential area consists of four curves: two vertical lines $\overline{GB}$ and $\overline{DE}$, the circular arc $\overline{EFG}$, and the elliptic arc $\overline{BCD}$. The view factor from the differential area to the cylinder can be determined by integrating over this contour [2]. Since the line integral over arc $\overline{BCD}$ is identical to that over the horizontal $\overline{BD}$, only the contour $\overline{BDEFGB}$ need be evaluated. The curves describing this contour can be expressed in the non-dimensional form as

$\overline{GB}$	$X = \sqrt{(P^2-1)}/P, \quad Y = 1/P, \quad Z = Z$
$\overline{DE}$	$X = -\sqrt{(P^2-1)}/P, \quad Y = 1/P, \quad Z = Z$
$\overline{DB}$	$X = X, \quad Y = 1/P, \quad Z = (P^2-1)/P \tan\theta$
$\overline{EFG}$	$X = \sin\beta, \quad Y = \cos\beta, \quad Z = H.$

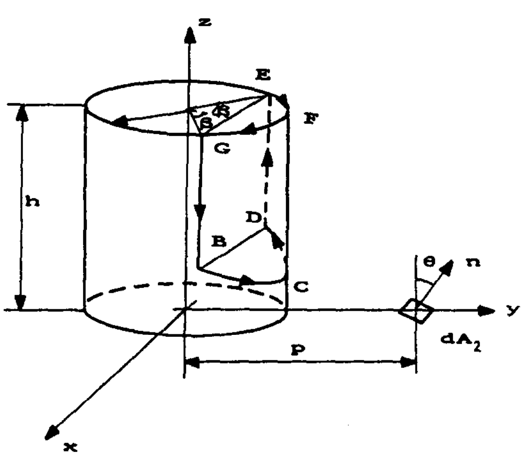

FIG. 1. Cylinder and differential element configuration.

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Integrating line $\overline{GB}$ from $Z = H$ to $(P^2-1)/P\tan\theta$, line $\overline{DB}$ from $X = \sqrt{(P_2-1)/P}$ to $-\sqrt{(P^2-1)/P}$, line $\overline{DE}$ from $Z = (P^2-1)/P\tan\theta$ to H, and arc $\overline{EFG}$ from $\beta = -\beta_0$ to β_0, where $\beta_0 = \cos^{-1}(1/P)$, yields the view factor from the differential area dA_2 to cylinder Cy

$$F_{dA_2-Cy} = \frac{1}{\pi}\left\{\tan^{-1}\left(\frac{\cos\theta}{\sqrt{(P^2-1)}}\right) - \frac{\sin\theta}{P}\left[\tan^{-1}\left(\frac{H}{\sqrt{(P^2-1)}}\right) - \tan^{-1}\left(\frac{\sqrt{(P^2-1)}}{P}\tan\theta\right)\right] + \frac{H\sin\theta}{2P} \times\left[\cos^{-1}(1/P) - 2\frac{1+P^2+H^2}{\sqrt{((1+P^2+H^2)^2-4P^2)}} \times\tan^{-1}\left(\sqrt{\left(\frac{(1+P)^2+H^2}{(1-P)^2+H^2}\right)}\tan\left(\frac{\cos^{-1}(1/P)}{2}\right)\right)\right] + \frac{\cos\theta}{2}\left[\frac{2(1-P^2-H^2)}{\sqrt{((1+P^2+H^2)^2-4P^2)}} \times\tan^{-1}\left(\sqrt{\left(\frac{(1+P)^2+H^2}{(1-P)^2+H^2}\right)}\tan\left(\frac{\cos^{-1}(1/P)}{2}\right)\right) + \cos^{-1}(1/P)\right]\right\}. \quad (1)$$

Equation (1), in the limiting case of $\theta = 0$, reduces to the solution given in ref. [3].

If $\tan^{-1}[HP/(P^2-1)] \leqslant \theta < \tan^{-1}[H/(P-1)]$, the view factor from the differential area to the cylinder will be identical to that from the differential area to the disk section traced out by $\overline{EFGE}$. The contours of the disk section are defined with β equal to $\cos^{-1}A$, where $A = (P-H/\tan\theta)$. This view factor can be expressed as

$$F_{dA_2-Cy} = \frac{1}{\pi}\left\{\tan^{-1}\left(\frac{\sqrt{(1-A^2)}}{P-A}\cos\theta\right) + \frac{H\sin\theta}{2P} \times\left[\cos^{-1}A - \frac{2(1+P^2+H^2)}{\sqrt{((1+P^2+H^2)^2-4P^2)}} \times\tan^{-1}\left(\sqrt{\left(\frac{(1+P)^2+H^2}{(1-P)^2+H^2}\right)}\tan\left(\frac{\cos^{-1}A}{2}\right)\right)\right] + \frac{\cos\theta}{2}\left[\frac{2(1-P^2-H^2)}{\sqrt{((1+P^2+H^2)^2-4P^2)}} \times\tan^{-1}\left(\sqrt{\left(\frac{(1+P)^2+H^2}{(1-P)^2+H^2}\right)}\tan\left(\frac{\cos^{-1}A}{2}\right)\right) + \cos^{-1}A\right]\right\}. \quad (2)$$

Figure 2 shows the variation of F_{dA_2-Cy} with angle θ for a number of P values. As expected, F_{dA_2-Cy} has a maximum value when the normal to the differential element is parallel to the axis of symmetry (i.e. at an angle $\theta = 0$), and drops off sharply as θ increases. At values of θ greater than $\tan^{-1}[H/(P-1)]$ the differential element will not see the cylinder, and thus F_{dA_2-Cy} will equal zero. As the distance P between the differential element from the axis of symmetry increases the view factor will decrease, since a smaller fraction of radiation leaving the element will reach the cylinder.

View factor from a cylinder to an axisymmetric body

A concise method has been developed for view factor calculations between specific axisymmetric bodies (sphere [1], disk [4], and cone [5]) and general coaxial axisymmetric bodies. This method can also be used to calculate the view factors between cylinders and coaxial axisymmetric bodies by employing equations (1) and (2).

Figure 3 shows the cylinder and a differential conical ring, formed by rotating the differential element about the axis of symmetry of the cylinder. Applying the reciprocity rule, noting the external surface area of the cylinder is $2\pi rh$, and integrating over the surface of the differential conical ring, the view factor from the cylinder to the ring can be expressed as

$$dF_{Cy-\delta r} = \int_0^{2\pi}\frac{y\,dl}{2\pi rh}F_{dA_2-Cy}\,d\phi = \frac{y}{rh}F_{dA_2-Cy}\,dl \quad (3)$$

where y is the distance from the ring to the axis of symmetry and dl the differential length of the ring.

An axisymmetric body can be generated by letting the distance y equal the functional description of the body $f(x)$, where x is the distance along the axis of symmetry. Noting that $dl = \sqrt{(1+f'(x)^2)}\,dx$, the view factor from the cylinder to the axisymmetric body can be found via a single integration

$$F_{Cy-B} = \frac{1}{rh}\int_{x_{min}}^{x_{max}} f(x)\sqrt{(1+f'(x)^2)}F_{dA_2-Cy}\,dx. \quad (4)$$

The limits of integration are defined as the extremal points

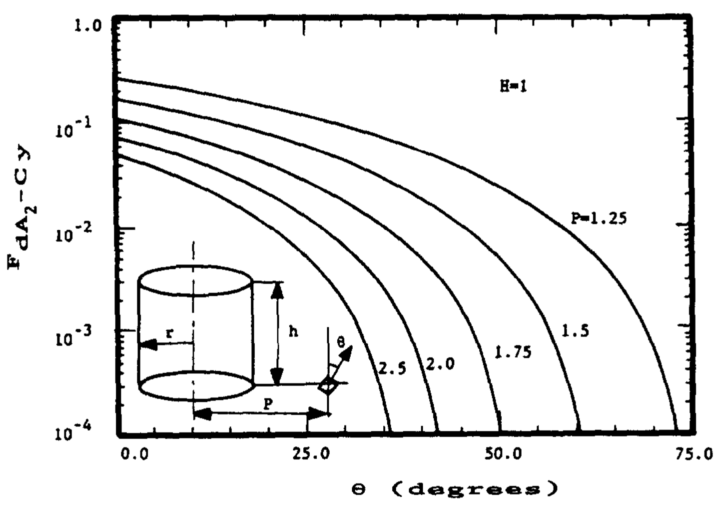

FIG. 2. Radiation view factor from a differential element dA_2 at an angle θ to the outer surface of a cylinder.

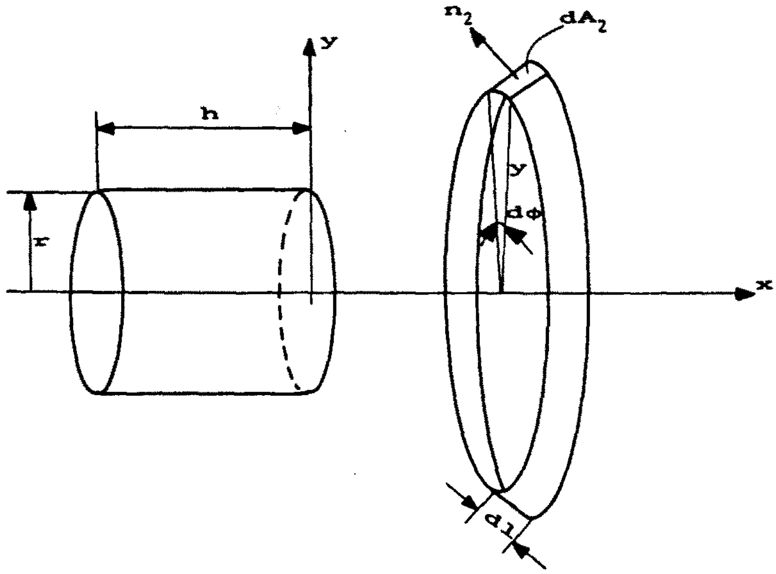

FIG. 3. Cylinder–coaxial differential conical ring geometry.

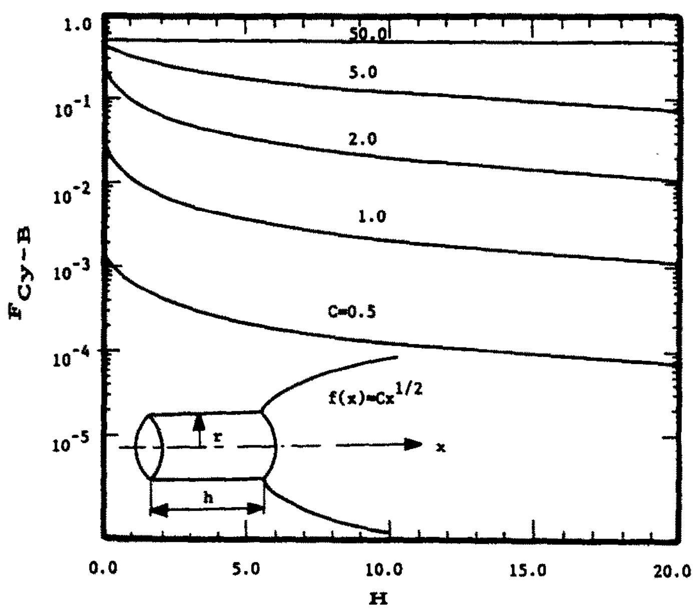

FIG. 4. Radiation view factor from a cylinder to a coaxial paraboloid.

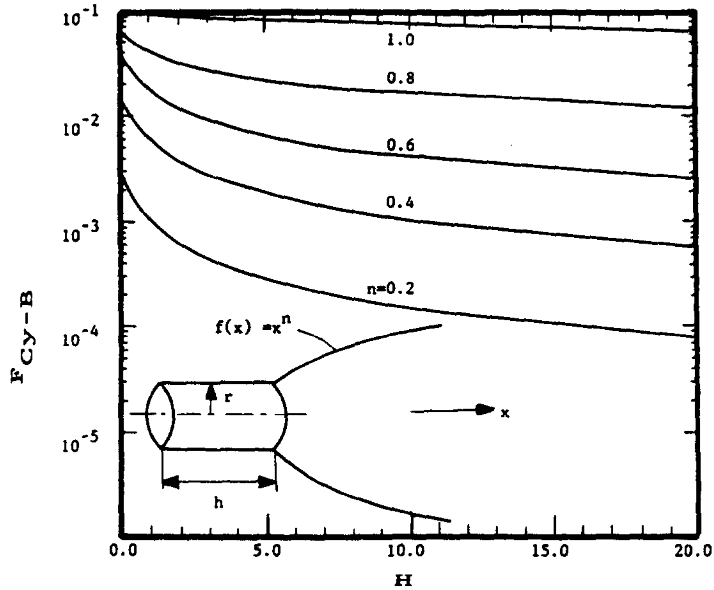

FIG. 5. Radiation view factor from a cylinder to a coaxial axisymmetric body generated via a power law.

where the axisymmetric body no longer sees the outer surface of the cylinder. They are dependent on the function generator $f(x)$, as well as the cylinder height and radius. Note, this formulation holds true only if there are no obstructions between any part of the axisymmetric body and the cylinder. To illustrate the usefulness of this approach, the view factors between cylinders and coaxial axisymmetric bodies formed via the power law equation, $f(x) = Cx^n$, will be examined.

First consider the case of an axisymmetric body in the shape of a paraboloid, where $n = 1/2$. Figure 4 shows the view factor as a function of the non-dimensional cylinder height H for a variety of coefficients C. The view factor decreases as H increases because a smaller fraction of radiant energy leaving the cylinder will be intercepted by the axisymmetric body. Since large values of C correspond to less concave paraboloids, view factors increase significantly as C increases. In the limiting case, as C approaches infinity, the paraboloid forms an infinite annular disk, for which the view factor equals 1/2.

Figure 5 illustrates the variation of the view factor with changes in the exponent n, where the coefficient C is kept at a constant value of unity. As the exponent increases towards a value of 1 (the case of a cone of slope C), the view factor increases. Again, this is because the concavity of the axisymmetric body decreases with increasing n.

CONCLUSION

Exact expressions for the radiation shape factor between a cylinder and a differential element of arbitrary orientation are derived. Based on these formulae a general method is described for calculating the view factor from the cylinder to a coaxial axisymmetric body using only a single numerical integration. The method is illustrated for axisymmetric bodies with function generators described by the power law equation. It is foreseen that such a procedure will be useful for radiation heat transfer calculations between cylindrical bodies and high density exhaust gases and between annular radiative fins and their bases.

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